Sustainability of public finances

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Contents

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- In the 1980s, 1990s, and 2000s several western countries faced increasing budgetary imbalances.

- In the European Union, the sustainability of public finances is a key feature in the Maastricht Treaty.

- To adhere to the Economic and Monetary Union countries need a budget deficit below 3% of GDP and a debt ratio below 60%.

- This fiscal framework is present in the Stability and Growth Pact, which strives for sustainable and sound fiscal positions, a necessary condition for the implementation of a single monetary policy.

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- Several studies have addressed the sustainability of public finances, assessing whether the intertemporal government budget constraint is fulfilled (see Afonso, 2005, Afonso, Rault, 2010, for an overview).

- Such assessments test empirically

    - for the <u>cointegration</u> between government revenue and government spending, or

    - for the existence of <u>non-stationarity</u> in the series of government debt and budget balances.

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- The government budget constraint, in real terms, not considering revenues from the issuance of money, is

$$B_t - B_{t-1} = r_t B_{t-1} + G_t - R_t \qquad (1)$$

G – government spending without interest payments;
R – government revenue;
B – stock of outstanding government debt;
r – real interest rate.

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$$B_t = (1 + r_t)B_{t-1} + G_t - R_t \qquad (2)$$

- The difference equation (2) relates the stock of government debt in t with the primary budget balance in t, plus interest payments stemming from t-1.

i) With balanced primary balances, $G_t = R_t$, the growth rate of government debt equals the real interest rate;

ii) With primary deficits, G_t-R_t>0, the growth rate of government debt is higher than the real interest rate;

iii) With primary surpluses, G_t-R_t<0, the growth rate of government debt is lower than the real interest rate.

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- Considering seigniorage revenues,

$$B_t = (1 + r_t)B_{t-1} + G_t - R_t - \frac{(M_t - M_{t-1})}{P_t} \qquad (3)$$

M – nominal monetary base;
P – price level.

- Using,

$$Y_t = (1 + y_t)Y_{t-1} \qquad (4)$$

Y – real GDP;
y – real GDP growth rate;

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- The government budget constraint as a ratio of GDP is

$$\frac{B_t}{Y_t} = \frac{(1+r_t)}{(1+y_t)}\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t}{Y_t} - \frac{R_t}{Y_t} - \frac{M_t - M_{t-1}}{P_t Y_t} \qquad (5)$$

with $b_t = B_t/Y_t$, $g_t = G_t/Y_t$, $\rho_t = R_t/Y_t$,

$$b_t = \frac{(1+r_t)}{(1+y_t)}b_{t-1} + g_t - \rho_t - \frac{M_t - M_{t-1}}{P_t Y_t} \qquad (6)$$

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$$M_t = (1 + \lambda_t)M_{t-1} \qquad (7)$$

- The government budget constraint (6), includes seigniorage, τ,

$$\tau = \frac{M_t - M_{t-1}}{P_t} \qquad (8)$$

$$\tau = \frac{M_t}{P_t}\frac{\lambda_t}{1 + \lambda_t} \qquad (9)$$

τ is the product of real monetary base (the base for the inflation tax) and the inflation tax (growth rate of monetary based divided by $(1+\lambda_t)$),

$$(1+\lambda_t) = (1+y_t)(1+\pi_t).$$

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- Seigniorage can also be measured as a % of GDP ($\Delta M/PY$).

- Recalling the money demand equation from the quantity theory of money,

$$\frac{M_t}{P_t} = kY_t \qquad (10)$$

$1/k$ – (constant) velocity circulation of money

- Seigniorage, as a ratio of GDP, τ^*, is derived from (9) and (10)

$$\tau^* = k\frac{\lambda_t}{1+\lambda_t} \qquad (13)$$

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$$\tau^* = k \frac{\lambda_t}{1 + \lambda_t} \qquad (13)$$

- Using (13) in the government budget constraint (6)

$$b_t = \frac{(1 + r_t)}{(1 + y_t)} b_{t-1} + g_t - \rho_t - k \frac{\lambda_t}{1 + \lambda_t} \qquad (14)$$

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- The government can sell assets (privatizations) *Z (z= Z /Y)*, to finance a budget deficit.

$$b_t = \frac{1+r_t}{1+y_t} b_{t-1} + g_t - \rho_t - k \frac{\lambda_t}{1+\lambda_t} - z_t \qquad (15)$$

- From (15), *ceteris paribus*, the growth rate of the debt-to-GDP ratio will decrease when the **real interest rate is lower than the real economic growth**.

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- A primary balance can be seen as sustainable if it provides a constant debt-to-GDP ratio:

$$b_t - b_{t-1} = 0 \qquad (16)$$

- The condition for the stability of the debt-to-GDP ratio can be derived from (15),
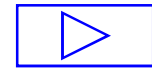
$$b_t - b_{t-1} = \frac{1+r_t}{1+y_t}b_{t-1} + g_t - \rho_t - k\frac{\lambda_t}{1+\lambda_t} - z_t - b_{t-1} \quad (17)$$

$$b_t - b_{t-1} = \frac{r_t - y_t}{1+y_t}b_{t-1} + g_t - \rho_t - k\frac{\lambda_t}{1+\lambda_t} - z_t \quad (19)$$

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$$b_t - b_{t-1} = \frac{r_t - y_t}{1 + y_t} b_{t-1} + g_t - \rho_t - k \frac{\lambda_t}{1 + \lambda_t} - z_t \qquad (19)$$

The development of the debt-to-GDP ratio depends of:

- the product between the accumulated stock of government debt and the difference between the real interest rate and the real growth rate;

- the primary budget balance;

- seigniorage revenues;

- revenues from selling assets.

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- One can recall the unpleasant monetarist arithmetic of Sargent and Wallace (1981)
- Since <u>when the real interest rate is above the real economic growth rate, the government needs to keep sufficient high enough primary surpluses</u>, which may prove difficult to achieve.

- From (16) and (19) the necessary primary budget balance that keeps the debt-to-GDP ratio constant is given by

$$\rho_t - g_t = \frac{r_t - y_t}{1 + y_t} b_{t-1} - k \frac{\lambda_t}{1 + \lambda_t} - z_t \qquad (20)$$

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- From (19), and in nominal terms,
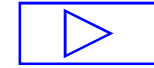
$$b_t - b_{t-1} = \frac{i_t - n_t}{1 + n_t} b_{t-1} + \left(g_t - \rho_t - k \frac{\lambda_t}{1 + \lambda_t} \right) \qquad (35)$$

- With the Fisher equation for the real interest rate, $r = i - \pi$, and for the nominal economic growth rate, $n = y + \pi$,

$$\Delta b = g - \rho - \lambda m + b(i - n) \qquad (36)$$

$$\Delta b = g - \rho - \lambda m + b(r + \pi - y - \pi) \qquad (37)$$

$$\boxed{\Delta b = g - \rho - \lambda m + b(r - y)} \qquad (38)$$

- From (38), debt follows an explosive path when $r > y$.

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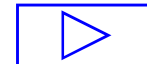- Assuming again $\Delta b=0$, (38) can also be solved for b,

$$b = (\rho - g + \lambda m)/(r - y) \qquad (39)$$

- The government budget constraint (33) can be simplified, omitting the monetary base,

$$\Delta b = g + ib - \rho - \lambda m - bn \qquad (33)$$
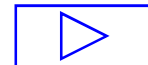
$$\boxed{\Delta b_t = d_t - nb_t} \qquad (40)$$

$d = g + ib - \rho$, budget deficit, % of GDP.

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- The 3% limit for the deficit in the Stability and Growth Pact of the EU, is an arbitrary value. The underpinnings go back to the beginning of the 1990s.

- A deficit of 3% of GDP is consistent with the long-run convergence of the debt ratio to 60%, assuming a nominal growth rate of 5% (for instance, 3% real growth and 2% inflation).
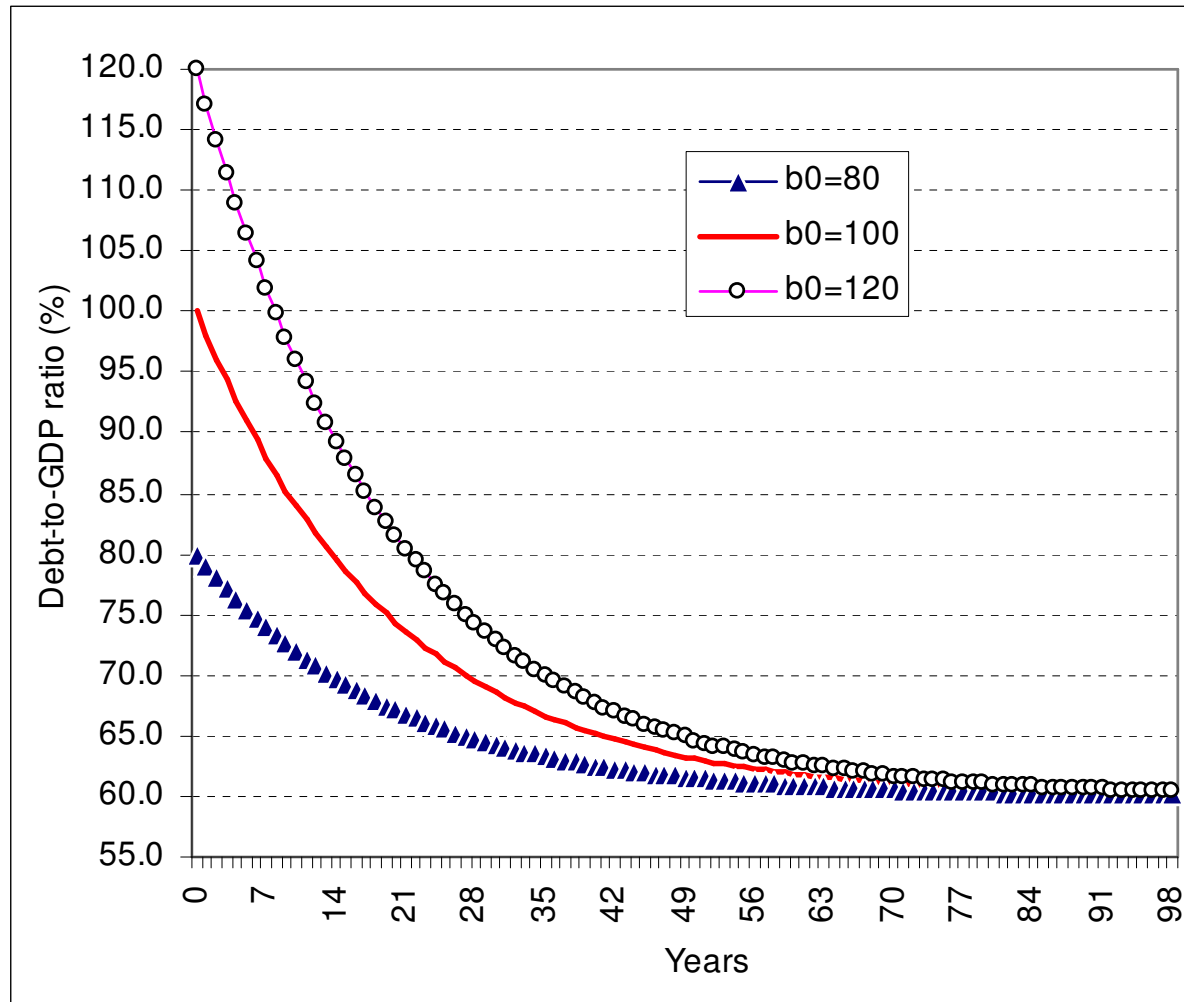
$$\Delta b_t = d_t - n b_t$$

- Trajectories for the debt-to-GDP ratio, for different starting debt ratio values: the ratio converges to $b=d/n$ (0.6=3/5), the limit of the series of the successive debt ratios.

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Debt-to-GDP trajectories (*n*=5%; *d*=3.5%)

$$\Delta b_t = d_t - nb_{t-1}$$

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Large deficits after the boom years and the crisis

General government budget balance (% of GDP)

	2000	2007	2010	2014		2000	2007	2010	2014
BE	-0,04	-0,05	-3,74	-2,61	MT	-5,72	-2,30	-3,54	-2,72
BG	-0,54	1,16	-3,12	-1,92	NL	1,97	0,18	-5,13	-3,19
CZ	-3,62	-0,74	-4,73	-2,81	AT	-1,68	-0,87	-4,51	-2,05
DK	2,26	4,81	-2,53	-1,26	PL	-3,03	-1,88	-7,89	4,97
DE	1,14	0,24	-4,17	-0,03	PT	-3,27	-3,15	-9,82	-4,04
EE	-0,23	2,39	0,18	-0,36	RO	-4,67	-2,91	-6,79	-2,16
IE	4,88	0,16	-30,61	-4,84	SI	-3,71	-0,05	-5,86	-3,95
EL	-3,73	-6,46	-10,68	-2,20	SK	-12,27	-1,81	-7,66	-3,26
ES	-0,94	1,97	-9,61	-5,78	FI	7,02	5,34	-2,49	-2,52
FR	-1,51	-2,73	-7,06	-4,03	SE	3,59	3,61	0,29	-1,49
HR			-6,36	-5,44	UK	3,53	-2,82	-10,07	-5,25
IT	-0,83	-1,63	-4,47	-2,57	**EA-17**	**-0,11**	**-0,67**	**-6,21**	**-2,57**
CY	-2,34	3,50	-5,29	-5,82	**EU-28**			-6,51	-2,70
LV	-2,79	-0,39	-8,14	-1,01					
LT	-3,20	-1,01	-7,18	-2,29					
LU	5,97	3,68	-0,81	-0,51					
HU	-3,04	-5,14	-4,31	-3,01					

Source: European Commission spring 2014 economic forecast,.

Notes: The Irish figure for 2010 includes significant one-off costs, largely related to measures to support the banking sector, which total around 19% of GDP. Excluding one-off costs, the budget balance would have been -13.2% of GDP for Ireland and -6.0% for the euro area.

3 - Arithmetic of the debt ratio (5)

Increasing government spending

General government spending (% of GDP)

	2000	2007	2010	2014		2000	2007	2010	2014
BE	49,04	48,20	52,45	53,94	MT	39,45	41,78	41,59	44,35
BG	41,27	39,19	37,42	38,65	NL	44,17	45,25	51,44	50,36
CZ	41,65	41,05	43,79	42,99	AT	51,78	48,47	52,82	51,76
DK	53,57	50,78	57,53	56,32	PL	41,08	42,18	45,43	41,26
DE	45,10	43,51	47,85	44,64	PT	41,57	44,30	51,47	46,82
EE	36,12	33,98	40,47	38,30	RO	38,56	38,24	40,10	36,07
IE	31,18	36,70	65,49	40,28	SI	46,51	42,28	49,45	49,29
EL	47,12	47,20	51,28	47,98	SK	52,14	34,21	39,95	37,73
ES	39,18	39,15	46,31	43,78	FI	48,34	47,39	55,48	57,83
FR	51,67	52,59	56,54	57,10	SE	55,09	50,93	52,03	52,38
HR			46,90	47,27	UK	36,42	43,29	49,88	45,97
IT	45,78	47,67	50,52	50,55	**EA-17**	**46,16**	**45,98**	**51,04**	**49,39**
CY	37,07	41,33	46,17	47,29	**EU-28**			50,58	48,55
LV	37,61	35,95	43,44	35,71					
LT	39,78	35,31	42,22	34,82					
LU	37,59	36,26	43,52	43,33					
HU	47,72	50,71	49,89	51,26					

Source: European Commission spring 2014 economic forecast,.

Notes: The Irish figure for 2010 includes significant one-off costs, largely related to measures to support the banking sector, which total around 19% of GDP.

- In the beginning of the 1920s, when writing about the public debt problem faced by France, Keynes (1923) highlighted the need for the French government to conduct a sustainable fiscal policy in order to satisfy its budget constraint, stating that the absence of sustainability would be evident when

"the State's contractual liabilities (…) have reached an excessive proportion of the national income" (Keynes, 1923, p. 54).

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- In modern terms, the sustainability of public finances is challenged when the government debt-to-GDP ratio reaches an excessive value. Government revenues are not enough to keep on financing the costs associated with the new issuance of public debt or, again in Keynes words, when

"it has become clear that the claims of the bond-holders are more than the tax payers can support" (p. 55). At that point the government will have to take measures that restore the sustainability of fiscal policy, meaning that the State "must come in due course to some compromise between increasing taxation, and diminishing expenditure, and reducing what (…) [it] owe[s]" (p. 59).

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- **Ponzi games**: in the 1920s Charles Ponzi swindled several investors in Boston, making interest payments to previous investors with the money received from new investors. He was arrested when was unable to pay his debts and keep the scheme alive.

- Theoretically, the government can also, to some extent, rollover existing government debt during several years.

- Such approach will be possible if the real interest rate is lower than the real economic growth rate ("honest Ponzi games", as labelled by Buiter, 1985).

- However, in the end, the government has to face its intertemporal budget constraint, and needs to repay its debt, delivering primary budget surpluses.

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- Fiscal policy sustainability can be assessed via the so-called present value borrowing constraint (PVBC). In GDP ratios:

$$\frac{B_t}{Y_t} = \frac{(1+r_t)}{(1+y_t)}\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t}{Y_t} - \frac{R_t}{Y_t} \qquad (53)$$

- Assuming the real interest rate to be stationary, with mean r, and considering also a constant real GDP growth, the budget constraint is then given by

$$b_{t-1} = \sum_{s=0}^{\infty}\left(\frac{1+y}{1+r}\right)^{(s+1)}\left[\rho_{t+s} - e_{t+s}\right] + \lim_{s \to \infty} b_{t+s}\left(\frac{1+y}{1+r}\right)^{(s+1)} \qquad (55)$$

with $b_t = B_t/Y_t$, $e_t = E_t/Y_t$ and $\rho_t = R_t/Y_t$.

$$e_t = g_t + (r_t - r)b_{t-1}$$

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- A sustainable fiscal policy needs to ensure that the present value of the stock of government debt, the 2nd term in the right hand side of (55), goes to zero in infinity,

- **Sustainability is more commonly assessed via cointegration analysis** between government revenue and spending (Hakkio and Rush, 1991). The implicit hypothesis concerning the real interest rate, with mean r, is also stationarity.

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-With the no-Ponzi game condition, spending and revenue must be cointegrated of order one for their first differences to be stationary.

- Assuming that spending and revenue are non-stationary variables, and that the first differences are stationary variables, this implies that the original series in levels are I (1).

- If spending and revenue are I(1), these two variables should be cointegrated with cointegration vector (1, -1) for the difference [spending-revenue] to be stationary.

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- Assessing the sustainability of the intertemporal government budget constraint involves testing the following cointegration regression (*R*-total revenue, *G*-total spending):

$$R_t = a + bG_t + u_t \qquad (65)$$

- If the null of no cointegration is rejected, then one should accept the alternative hypothesis of cointegration (u_t must be stationary).

- If *G* and *R* are non-stationary variables in levels, *0 < b < 1* is sufficient for the budget constraint to be obeyed (Hakkio and Rush (1991)).

- When revenues and expenditures are expressed as a *% of GDP* (or in per capita terms), it is necessary to have *b = 1* for the trajectory of the debt ratio not to diverge in an infinite horizon.

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- Single country cointegration analysis has been the widely used to assess fiscal sustainability (see Afonso (2005) for survey and EU results).

- **Panel data methods have also recently been used to assess fiscal sustainability**, testing for cointegration between government expenditure and revenue (Westerlund and Prohl (2010) for OECD countries, and Afonso and Rault (2010, 2015), Afonso and Jalles (2014), for the EU).

- Using a panel approach increases the power that may be brought to the cointegration hypothesis (increased number of observations that results from adding the individual time series).

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- Even if there is no single fiscal policy in the EU, panel analysis of the sustainability of public finances is relevant in the context of 28 EU countries seeking to pursue sound fiscal policies within the framework of the Stability and Growth Pact.

- Cross-country dependence can be envisaged either in the run-up to EMU or, for example, via integrated financial markets.
- Cross-country spillovers in government bond markets are to be expected, and interest rates comovements inside the EU have also gradually become more noticeable.

- Testable cointegration relation:

$$R_{it} = \alpha_i + \beta_i G_{it} + u_{it} \quad i=1,\dots,N;\; t=1,\dots,T \qquad (66)$$

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EU22 panel (max. time span, 1960-2006)

Country	β	t-stat	α	t-stat
Austria	0.796	31.840	6.829	7.128
Belgium	0.579	11.580	4.856	2.382
Cyprus	0.909	23.921	−6.466	−4.054
Czech Republic	0.834	25.273	−4.135	−2.163
Denmark	0.968	13.829	−0.425	−0.297
Finland	0.671	6.156	8.828	3.544
France	0.385	3.438	19.355	5.461
Germany	0.956	14.708	3.525	2.116
Greece	0.464	4.377	3.044	2.955
Ireland	0.517	3.059	6.251	0.918
Italy	0.432	6.000	2.768	1.113
Latvia	0.908	17.804	−4.181	−1.808
Lithuania	0.808	25.250	−5.361	−3.630
Luxembourg	0.999	12.183	0.871	0.377
Malta	0.900	56.250	0.883	1.079
Netherlands	0.926	28.061	−0.094	−0.075
Portugal	0.590	6.629	0.108	0.121
Romania	0.848	35.333	−3.117	−2.673
Slovenia	0.913	65.214	−1.188	−1.256
Spain	0.864	15.158	1.643	1.996
Sweden	0.651	11.224	16.165	7.600
UK	0.737	4.494	7.960	1.415

CCE common correlated effect estimation procedure. (Table 4c, Afonso and Rault, 2015).

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Evidence:

- β is always statistically significant, and with the right sign, its magnitude is also below unity.

- The size of the β is quite high in some cases and above, for instance, 0.8, notably for Denmark, Germany, Luxembourg, and the Netherlands (public finances may have been relatively less unsustainable in the past for the abovementioned four countries).

- Lower magnitude of the estimated β coefficient for several countries such as Belgium, Greece, Ireland, Italy, or Ireland (reflects a bigger departure from a one-to-one linkage between expenditures and revenues in the cointegration relationship).

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Background

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- Government budget constraint in nominal terms (omitting for simplicity privatization revenues),

$$B_t = (1 + i_t)B_{t-1} + G_t - R_t - \left(M_t - M_{t-1}\right) \qquad (21)$$

$$B_t - B_{t-1} = i_t B_{t-1} + G_t - R_t - \left(M_t - M_{t-1}\right) \qquad (22)$$

without subscripts,

$$\Delta B = G - R + iB - \Delta M \qquad (23)$$

- To assess the path of the debt-to-GDP ratio we need to compute the total derivative of B/Y:

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$$d\left(\frac{B}{Y}\right) = \frac{\partial(B/Y)}{\partial B}dB + \frac{\partial(B/Y)}{\partial Y}dY \qquad (24)$$

$$d\left(\frac{B}{Y}\right) = \frac{1}{Y}dB + \left(-\frac{B}{Y^2}\right)dY \qquad (25)$$

$$d\left(\frac{B}{Y}\right) = \frac{dB}{Y} - \frac{B}{Y}\frac{dY}{Y} \qquad (26)$$

- With a notation similar to (23), for small changes,

$$\Delta\left(\frac{B}{Y}\right) = \frac{\Delta B}{Y} - \frac{B}{Y}\frac{\Delta Y}{Y} \qquad (27)$$

using $b = B/Y$, $n = \Delta Y/Y$, (27) becomes

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$$\Delta b = \frac{\Delta B}{Y} - bn \qquad (28)$$

Substituting (23) in (28),

$$\Delta b = \frac{G + iB - R}{Y} - \frac{\Delta M}{Y} - bn \qquad (29)$$

with $g = G/Y$, $\rho = R/Y$,

$$\Delta b = g + ib - \rho - \frac{\Delta M}{Y} - bn \qquad (30)$$

$\lambda = \Delta M/M$, growth rate of the monetary base; $m = M/Y$,

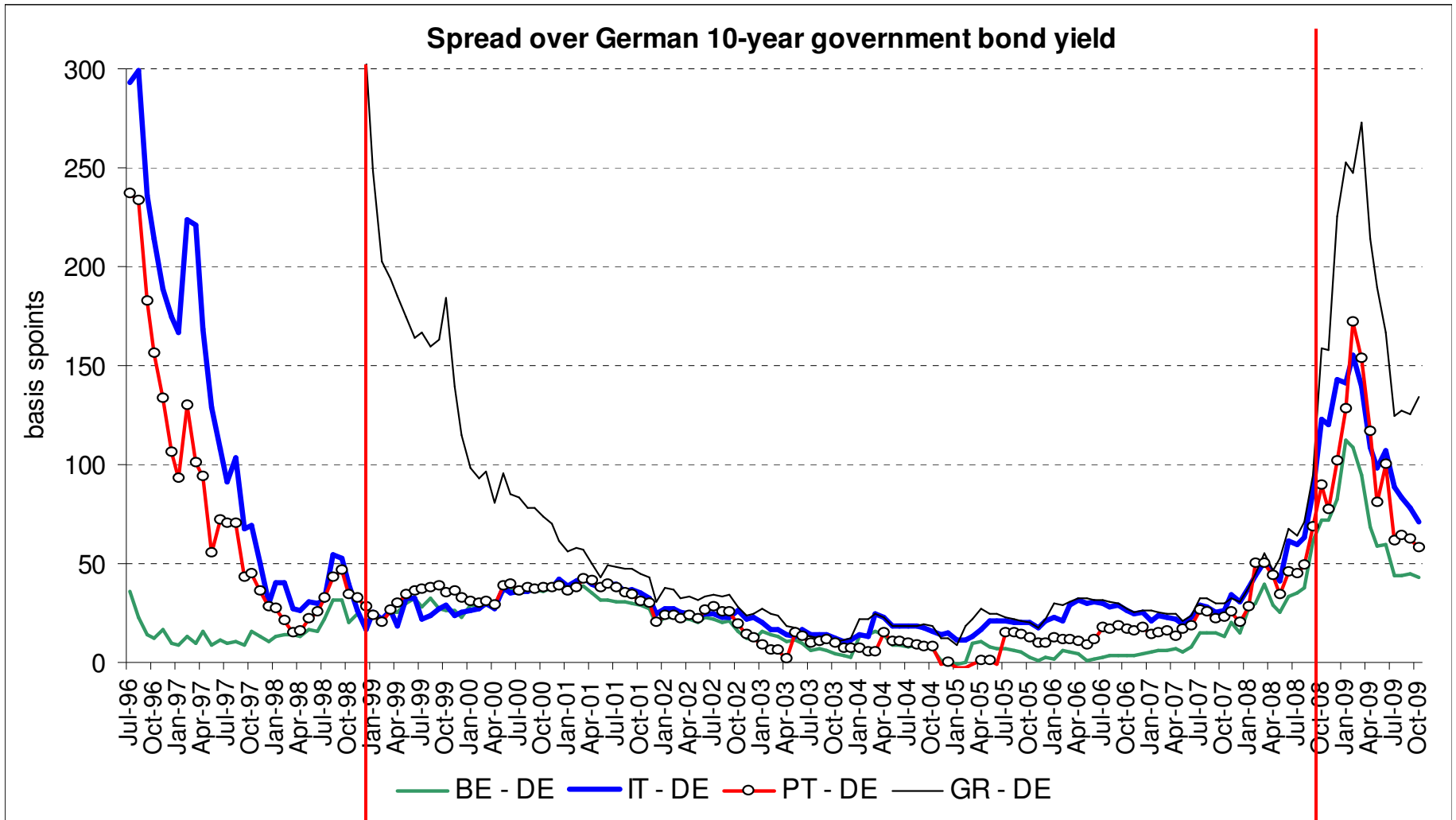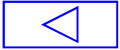$$\frac{\Delta M}{M} \frac{M}{Y} = \lambda m \qquad (32)$$

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with (32) and (30),

$$\Delta b = g + ib - \rho - \lambda m - bn \qquad (33)$$

- The long-term government budget constraint implies $\Delta b = 0$,

$$nb = g + ib - \rho - \lambda m \qquad (34)$$

- From (34), in the long-run, the budget deficit is approximately proportional to the level of government debt, and the proportionality factor is the real economic growth rate.
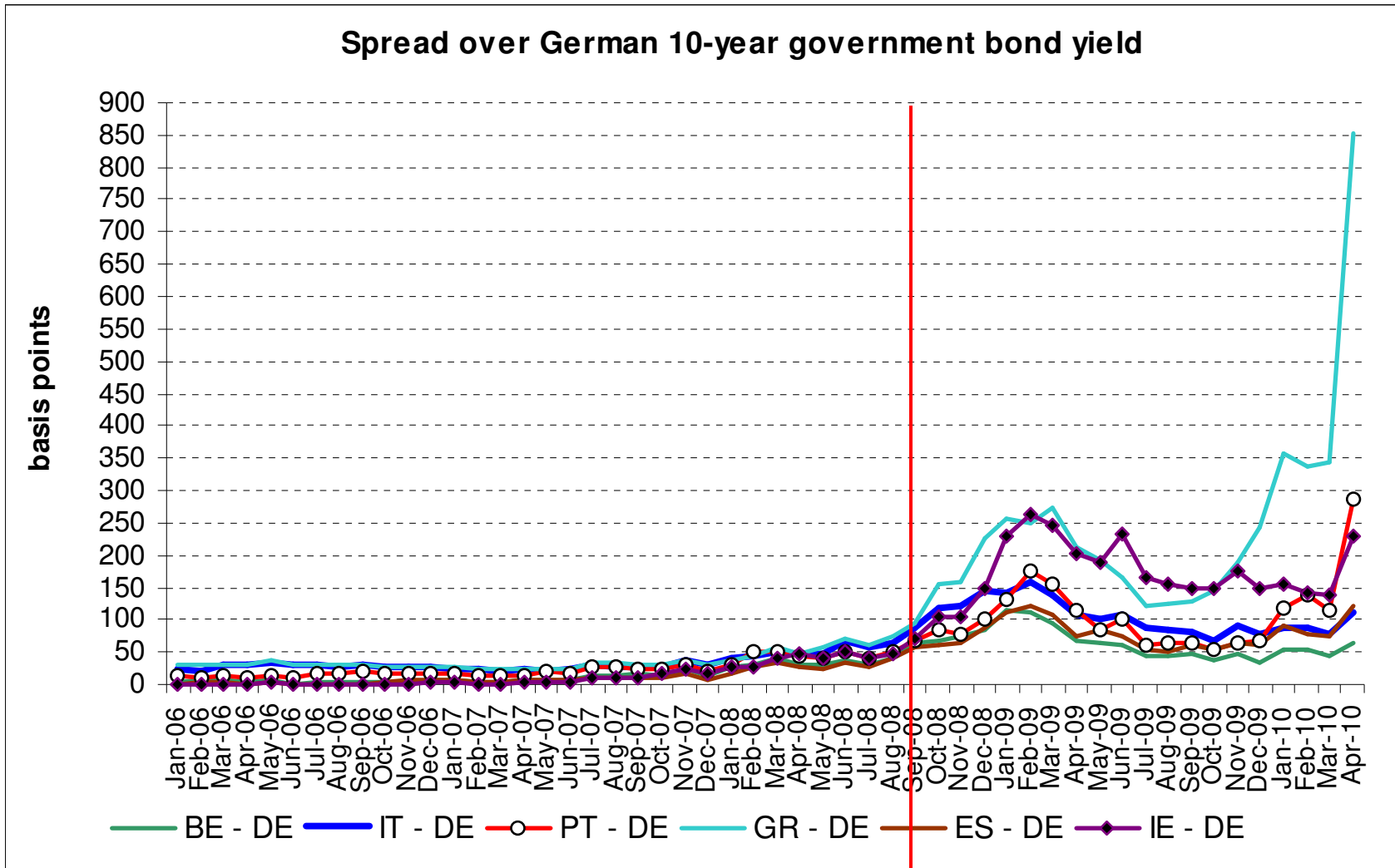
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Spread over German 10-year government bond yield

Long-term interest rates 1/
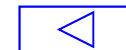
Sources: Reuters.

Jan. 1999, launch of the euro.

Lehman Brothers filed for bankruptcy protection on September 15, 2008.

Legend: BE - DE, IT - DE, PT - DE, GR - DE

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Spread over German 10-year government bond yield

Sources: Reuters.

Lehman Brothers filed for bankruptcy protection on September 15, 2008.

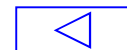A. Afonso

- Assuming balanced budgets $d_t=0$,

$$\dot{b}_t = -nb_t \qquad (41)$$
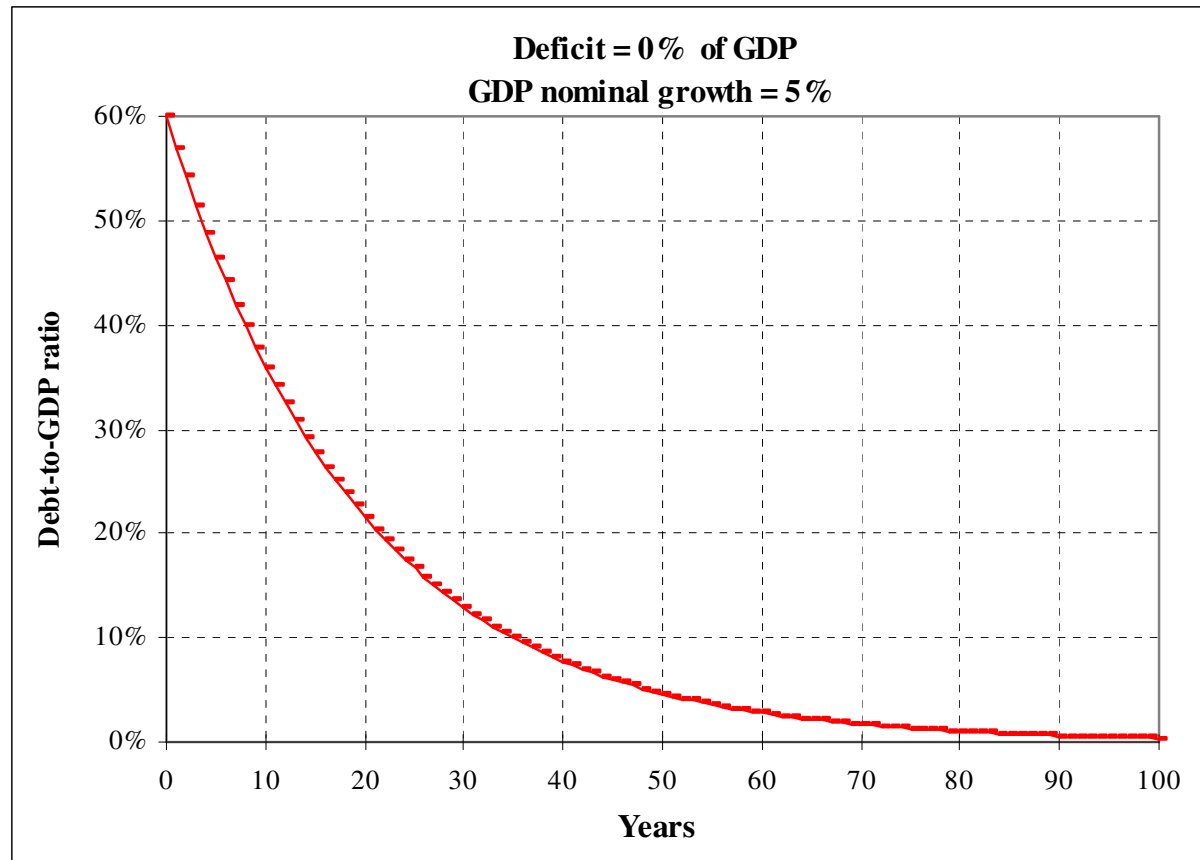
- The solution for the differential equation (41) is

$$b_t = Ce^{-nt} \qquad (42)$$

which converges to zero in the long-run since $\lim_{t \to \infty} \left[Ce^{-nt} \right] = 0$.

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Simulation of path for the debt-to-GDP ratio

Deficit = 0% of GDP
GDP nominal growth = 5%

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$$\Delta b_t = d_t - nb_{t-1},$$ (1)

fixing d and n,

$$b_t = d + (1-n)b_{t-1},$$
$$b_{t+1} = d + (1-n)d + (1-n)^2 b_{t-1},$$
$$b_{t+2} = d + (1-n)d + (1-n)^2 d + (1-n)^3 b_{t-1},$$
$$b_{t+k} = d[1 + (1-n) + (1-n)^2 + ... + (1-n)^k] + (1-n)^{k+1} b_{t-1}.$$ (2)

Since (1-n) <1, then $\lim_{k\to\infty}(1-n)^{k+1} b_{t-1} = 0$.

The right-hand side of (2) is d times the content of the square brackets, a geometric series, with the constant multiplier *(1-n)*.

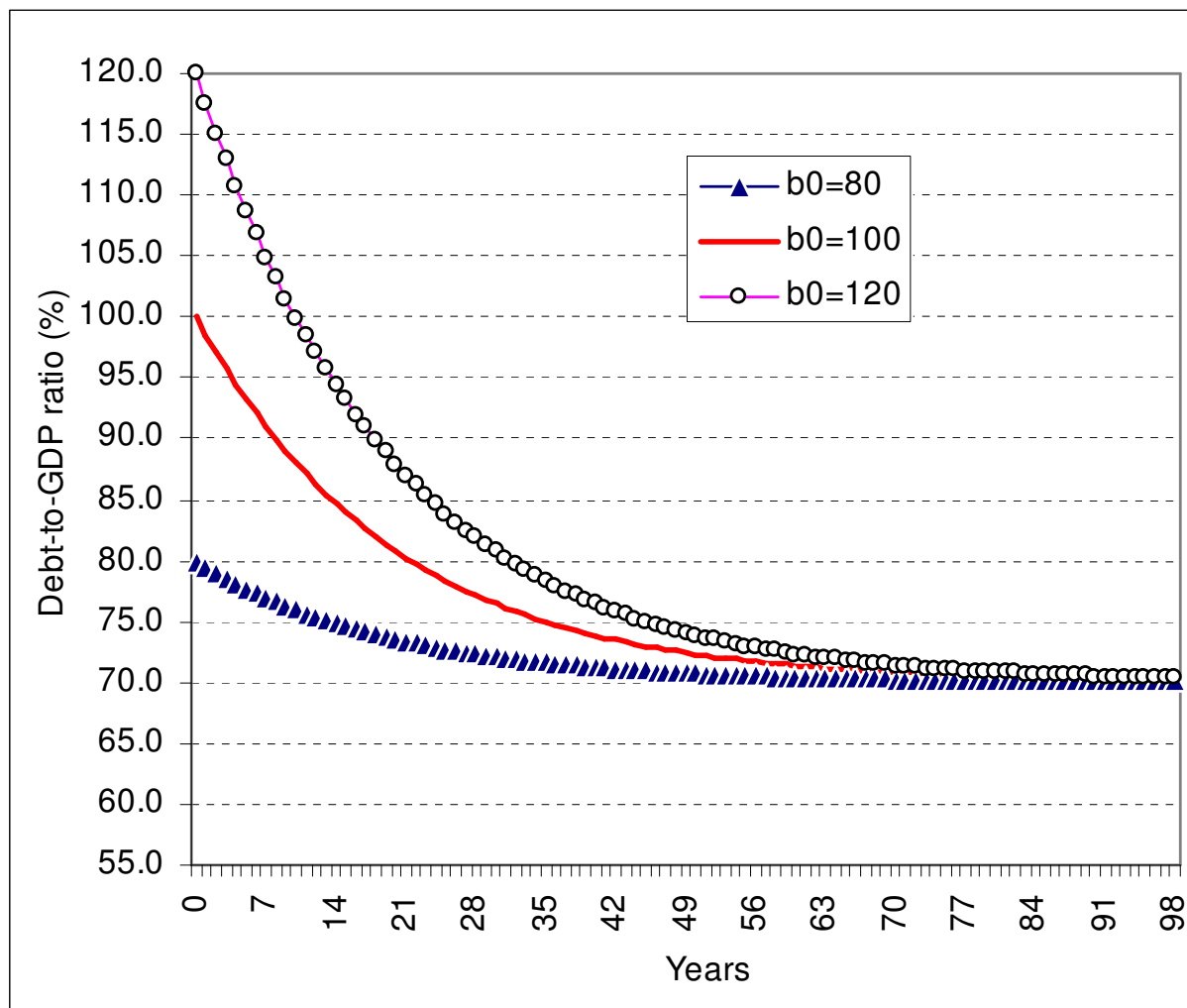Since its sum is $S = \dfrac{a_1 - a_n r}{1-r}$, then $S_k = \dfrac{1-(1-n)^{k+1}}{1-(1-n)}$.

With $\lim_{k\to\infty}(1-n)^{k+1} = 0$, then $\lim_{k\to\infty} S_k = 1/n$, and the content of the square brackets in (2) is *(1/n)*.
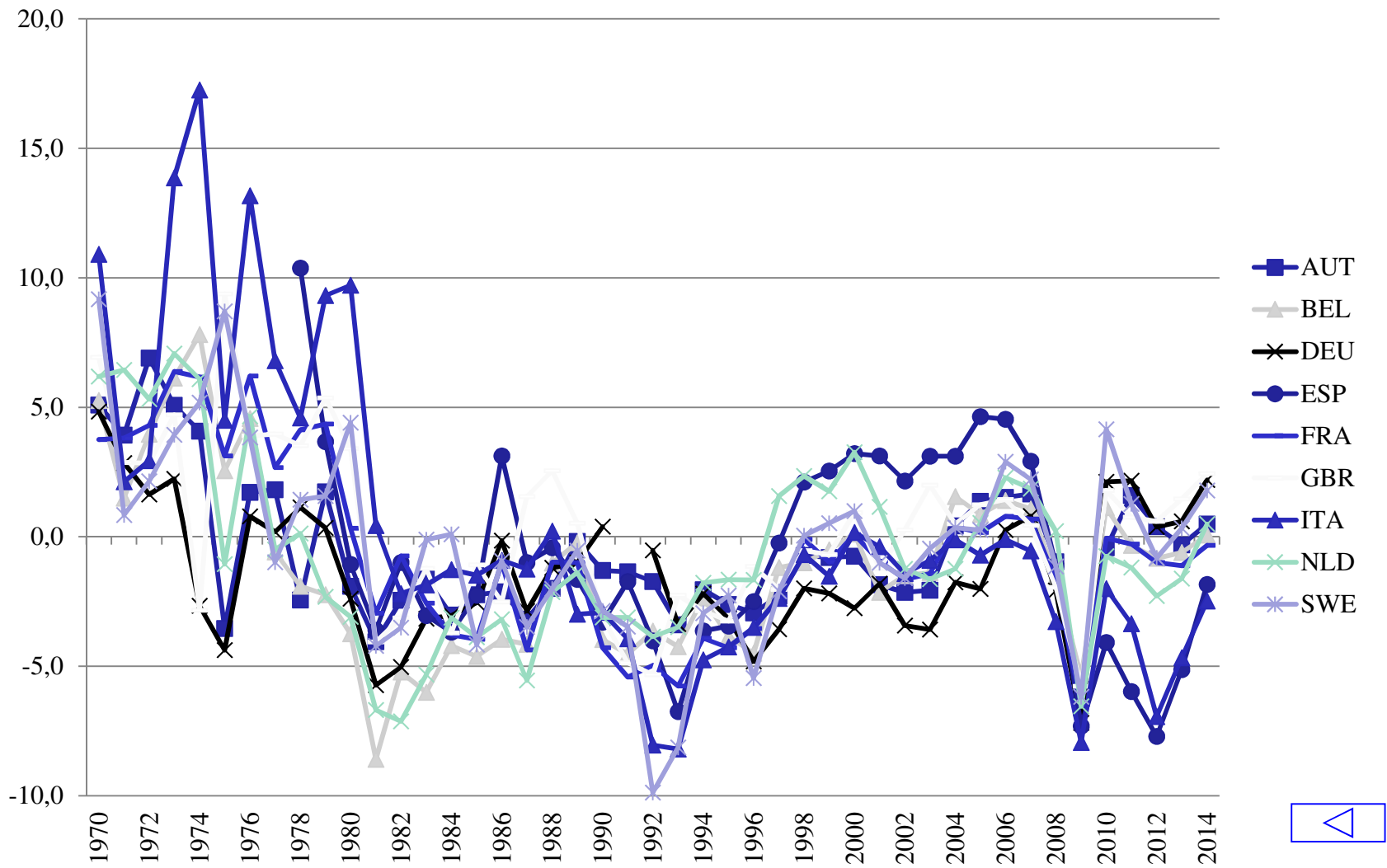
Therefore, b converges to d/n.

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Debt-to-GDP trajectories (n=5%; d=3.5%)

$$\Delta b_t = d_t - nb_{t-1}$$

Debt ratio convergence

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$$\Delta b = g - \rho - \lambda m + b \ (r\text{-}y)$$

Nominal growth rate – nominal interest rate

Legend: AUT, BEL, DEU, ESP, FRA, GBR, ITA, NLD, SWE

Source: Ameco, April 2015.

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$$\Delta b = g - \rho - \lambda m + b\ (r\text{-}y)$$

Nominal growth rate – nominal interest rate

PRT
DNK
FIN
GRC
IRL
CAN
JAP
USA

Source: Ameco, April 2015.

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